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14. ABSTRACT
Moments of inertia (MOI) are fundamental mass properties that provide information on a vehicle's mass distribution. The properties impact vehicle design and safety and are primary inputs to vehicle dynamics and mobility computer models, where MOI data for a vehicle's total, sprung, and unsprung masses are often required. Moments of inertia are also useful in the design and construction of vehicle safety outriggers needed during the conduct of dynamic handling tests. This document describes practical methods for the determination of the mass MOI of large, heavy military vehicles, typically about the centroidal axes.

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U.S. ARMY TEST AND EVALUATION COMMAND
TEST OPERATIONS PROCEDURE

*Test Operations Procedure 01-2-520 — 3 August 2017
DTIC AD No.

MOMENTS OF INERTIA

1. SCOPE.

a. This document describes practical methods for the determination of the mass moments of inertia (MOI) of large, heavy military vehicles, typically about the centroidal axes. Based on the Society of Automotive Engineers (SAE) vehicle coordinate system[1**], the MOI measurements described in this procedure include I_{xx} (roll), I_{yy} (pitch), I_{zz} (yaw), and I_{xz}.

b. The MOI's are fundamental mass properties that provide information on a vehicle's mass distribution. The properties impact vehicle design and safety and are primary inputs to vehicle dynamics and mobility computer models, where MOI data for a vehicle's total, sprung, and unsprung masses are often required. Moments of inertia are also useful in the design and construction of vehicle safety outriggers needed during the conduct of dynamic handling tests.

2. FACILITIES AND INSTRUMENTATION.

a. Measurement of the MOI's is typically performed using a compound pendulum, and may be challenging in practice due to the wide variety of large military vehicles under test. Military vehicles and trailers vary significantly in terms of weight, size, number of axles, and use of track versus wheels. Typical ranges include:

(1) Weight - 5000 to 150,000 pound force (lbf).

(2) Length - 180 to 480 inches (in.).

(3) Width - 90 to 144 in.

(4) No. of axles (wheeled vehicles) - up to five.

(5) Center of gravity (CG) height above ground - 30 to 70 in.

b. Because of these variations, flexibility of the MOI facility's layout is an important attribute.

2.1 Facility Descriptions.

a. Background. Methods for measuring the moments of inertia of large vehicles are relatively limited, but multiple advances in technology and design have occurred over the past 40 years. The Highway Safety Research Institute (HSRI) Pitch Plane Inertial Test Facility[2] used a hanging pendulum with an aluminum truss structure to support pitch (I_{yy}) inertia measurements of two- or three-axle vehicles up to 25,000 pounds (lbs). Axles were not located more than 12 feet longitudinally from the vehicle's CG. The HSRI facility also used a separate hanging pendulum for roll inertia (I_{xx}) measurements and a large horizontal hydrostatic bearing with linear springs beneath the vehicle for yaw inertia measurements.

** Superscript numbers correspond to Appendix D, References.

(1) The National Highway Traffic Safety Administration's (NHTSA) Vehicle Research and Test Center developed the Inertial Parameter Measurement Device (IPMD) for measuring the pitch and roll inertia and CG height via a hanging swing. A 37-inch diameter table bearing with springs was attached to the vehicle platform to measure the yaw inertia[3]. The IPMD was limited to passenger vehicles and light trucks ranging from 1000 to 6000 lbs with a maximum CG height of 30 in. above the ground due to the fixed pivot height. The Vehicle Inertia Measurement Facility (VIMF) was subsequently designed as an improvement to the IPMD by increasing its load capacity to 10,000 lbs, while making the support platform much more rigid[4,5]. The roll inertia was measured via an inverted pendulum with a roll-restoring spring. The VIMF also added the ability to accurately measure the roll-yaw (I_{xz}) product of inertia by kinematically linking the platform's roll and yaw motions.

(2) The U.S. Army Tank Automotive Research, Development and Engineering Center (TARDEC) Vehicle Inertial Properties Evaluation Rig (VIPER) was developed to measure the inertial properties of a wide range of military vehicles from 3,000 to 60,000 lbs, up to 120 in. wide, and with the farthest axle being up to 195 in. from the vehicle CG[6]. The VIPER used a multi-piece platform which could be configured depending on the size of vehicle under test. The CG height and pitch and roll inertia tests were conducted with the platform in a stable pendulum configuration with adjustable-radius curved rails used to provide the pivot. The yaw inertia was measured by rotating the platform on a turntable bearing with yaw spring constraints, while the roll-yaw product of inertia was measured simultaneously using a rigid roll-axis load cell. The Vehicle Inertia Parameter Evaluation Rig (VIPER II) expanded upon the VIPER's capabilities, as described in paragraph 2.1.c[7,8]. In general, it's apparent that facility capacities are increasing while measurement errors are decreasing in an effort to keep up with the Modelling and Simulation community's demand for more and higher accuracy inputs.

b. The U.S. Army Aberdeen Test Center (ATC) Facility. The ATC MOI facility (see Figure 1) consists of a large, solid-decked steel beam platform forming a reconfigurable stable (hanging) pendulum to measure the pitch, roll, and yaw moments of inertia of heavy, large vehicles. Drive-on capability is provided via hydraulic actuators that lower the platform to the ground for loading. The hydraulic actuators then raise the platform into position for attachment of the adjustable-length pendulum suspension links. The suspension links are connected overhead to a static support frame rigidly mounted to the ground with steel columns. Overhead pivot supports with low friction roller bearings provide rotation in pitch and roll. The suspension links are rearranged from the platform sides to the ends to change the rotation axis. For yaw measurements, the suspension links are replaced with vertical cables at the platform's corners to form a multi-filar torsional pendulum. Platform excitation is provided by manual force application with gravity providing the restoring force. Table 1 provides a summary of the facility's capabilities.

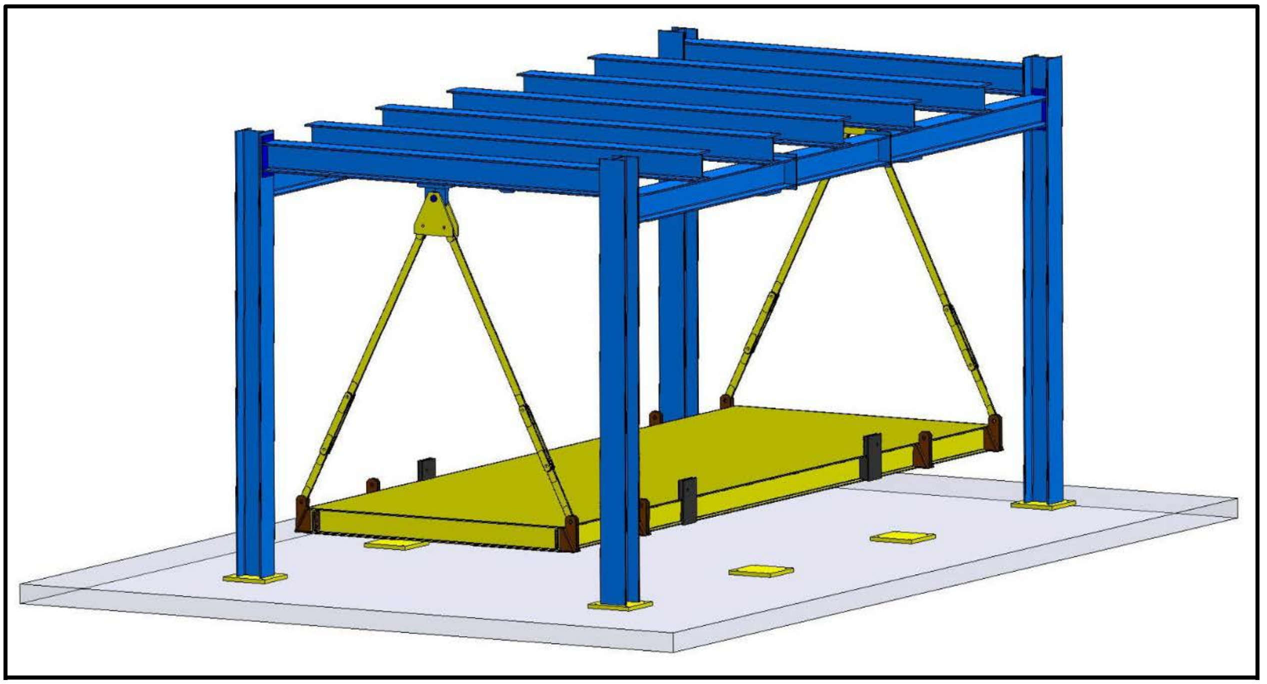

Figure 1. ATC Large Platform MOI Facility configured for generic roll test. (Some support structures have been removed from the drawing for clarity.)

TABLE 1. ATC MOI FACILITY LARGE VEHICLE PLATFORM CAPABILITIES

PARAMETER	SPECIFICATION	
	SI units	English units
Minimum Vehicle Mass[a]	Approx. 8390 kilograms (kg)	Approx. 18,500 pound mass (lbm)
Maximum Vehicle Mass	63,500 kg	140,000 lbm
Maximum Number of Axles	Unlimited	
Track Width (outside-to-outside)	Up to 378 centimeters (cm)	Up to 149 in.
Maximum Practical Vehicle Width	373 cm	147 in.
Maximum Practical Vehicle Length	Approx. 1520 cm	Approx. 600 in.
Maximum Longitudinal Distance from Vehicle CG to Farthest Axle	549 cm	216 in.
Maximum Vehicle CG Height	Approx. 330 cm	Approx. 130 in.

[a]Dependent on facility component uncertainties and maximum allowable overall error.

c. The TARDEC Facility. The VIPER II facility (see Figure 2) utilizes three different vehicle platforms to measure the vehicle's CG height, pitch, roll, and yaw moments of inertia, and roll-yaw product of inertia, depending on vehicle size. CG height and pitch MOI are

measured via a stable pendulum with adjustable pivot locations. A Yaw Base, containing the roll axis bearings and the yaw axis bearing, is positioned beneath the vehicle Platform during the roll and yaw moment of inertia tests and roll-yaw product of inertia (I_{xz}) test. The roll MOI is measured using an unstable (or inverted) pendulum approach, stabilized with linear springs with the pivot fixed under the platform. The yaw MOI is measured using a torsional pendulum approach, again with linear restoring springs. During the yaw MOI measurement, a load cell constrains the roll axis to allow calculation of the roll-yaw product of inertia (I_{xz}). Table 2 provides a summary of the facility's capabilities.

Figure 2. TARDEC VIPER II Facility configured for a yaw MOI test.

TABLE 2. TARDEC MOI FACILITY VIPER II CAPABILITIES[8]

PARAMETER	SPECIFICATION	
	SI units	English units
Minimum Vehicle Mass	1360 kg	3000 lbm
Maximum Vehicle Mass	45,360 kg	100,000 lbm
Maximum Axle Static Load	133,450 Newtons (N)	30,000 lb
Maximum Number of Axles	Unlimited	
Maximum Turret Weight	27,216 kg	60,000 lbm
Minimum Track Width	140 cm	55 in.
Maximum Track Width	300 cm	118 in.
Maximum Vehicle Width	381 cm	150 in.
Maximum Tire Width	81 cm	32 in.
Maximum Vehicle Length	1520 cm	600 in.
Maximum Longitudinal Distance from Vehicle CG to Farthest Axle	635 cm	250 in.

2.2 Instrumentation.

a. Allowable Measurement Uncertainty. The permissible measurement errors listed in Table 3 were established to minimize the test method's controllable parameter errors.

TABLE 3. PERMISSIBLE MEASUREMENT ERRORS

MEASURED PARAMETER	PERMISSIBLE MEASUREMENT ERROR	
	SI units	English units
Time	0.001 seconds (sec)	
Length	1.0 millimeter (mm)	0.04 in. (0.003 ft)
Displacement	0.25 mm	0.01 in. (0.0008 ft)
Weight, Force	0.25 percent of reading	
Angle	0.01 degrees	
Gravity constant	0.0001 meters per second squared (m/s^2)	0.004 inches per second squared (in/s^2) (0.0003 feet per second squared (ft/s^2))

Note: The errors listed above are all 1-sigma standard deviation values for a normal distribution.

b. Descriptions.

(1) Oscillation Period. The pendulum's period of oscillation may be measured using a number of devices, including a digital period counter connected to a photoelectric or induction timing sensor. Inclinometers and angular rate sensors connected to data acquisition systems are also effective, as are optical angle encoders attached at the pivot axis. A six degree-of-freedom (DOF) Inertial Motion Unit (IMU) sensor is useful for multi-axis measurements to ensure pure rotational motion is present during yaw testing with a multi-filar pendulum. Inclinometers, angular rate sensors, optical angle encoders and IMUs enable the calculation of system damping and observation of angular accelerations needed for product of inertia (POI) determination. A potential shortcoming of commercial IMUs is a relatively low data rate output (typically a maximum of 100 Hz).

(2) Length and Displacement. Typically a steel tape with 1 mm divisions is used for vehicle dimension measurements. Laser range finders or ultrasonic sensors may be useful for pivot height and length of multi-filar suspension link measurements with respect to the platform. Ultrasonic or optical transducers are recommended for vehicle displacement with respect to the platform.

(3) Weight. A platform scale or individual wheel scales are typically used prior to MOI testing to measure total vehicle weight. If the total vehicle weight is calculated by multiple measurements of individual axle weights, measurement uncertainty may be increased. A tension load-cell and lifting device may also be used to weigh the entire vehicle or measure the facility

platform weight. Smaller capacity scales are typically used to weigh vehicle-to-platform restraint items.

(4) Force and Torque. A force or torque sensor (load cell) is used to measure the reaction force or moment necessary to calculate the product of inertia I_{xz}. The sensor must be sufficiently stiff in the measurement direction. It is recommended that the constrained system natural frequency with the load cell is above 50 Hz, so as to not influence the platform motion by introducing an unwanted degree of freedom[5]. This sensor is only needed when measuring POI's.

(5) Digital Data Acquisition System. It is recommended that the measurements are recorded with a digital data acquisition system for accurate analysis. At a minimum, a 1000 Hz sampling rate shall be used when recording the platform oscillation period. The selected sample rate will affect the choice of low-pass filter cut-off frequencies.

c. Sensor Locations.

(1) Oscillation Period. Placement of an oscillation period sensor is dependent on the sensor used. When using optical or inductive timing sensors, it is recommended to place the sensor at the mid-point of the oscillatory motion. For yaw oscillations with a torsional pendulum, the use of three evenly spaced timing locations may improve measurement accuracy by averaging timing differences due to platform levelness errors[9]. Optical angular encoders should be mounted coincident to the pivot axis. IMU axes shall be oriented parallel to the vehicle and platform motion coordinate axes. It is recommended that the IMU is located on the platform's centerline for pitch/roll measurements and on the platform's centroidal yaw axis for yaw measurements. Any inclinometers used should be mounted in such a way as to eliminate any loaded platform deflection effects.

(2) Displacement. For I_{yy} (pitch) measurements, it is recommended that two vehicle displacement sensors be used to dynamically measure longitudinal movement of the vehicle during testing. The sensors should be located on either end of the vehicle in the longitudinal direction, equidistant from the vehicle's longitudinal centerline so their outputs can be averaged. For the roll orientation (I_{xx}), it is recommended to use two sensors located on the same side of the vehicle oriented laterally. Again, the sensors should be positioned equidistant from the vehicle's center of gravity so their output can be averaged. For the yaw orientation (I_{zz}), the sensors can remain in their roll displacement locations, where the difference in their outputs is used with the distance between them to find the vehicle's displacement angle. In all cases, the displacement sensors should be mounted parallel to the platform surface at the vehicle's CG height to ensure only translational motion is recorded. Care should be taken to ensure the displacement measured is of an integral vehicle component (e.g., the body) and not of a part that can move independently of the vehicle as a whole (e.g., a loose door). Care should also be taken to ensure the sensor mounts are properly stiff to avoid uncontrolled movement of the sensor during the test.

(3) Force and Torque. When a product of inertia (I_{xz}) is to be determined, the sensor(s) used to measure the reaction moment of the constrained roll axis during I_{zz} (yaw) tests shall be sufficiently located to ensure the moment measurement is parallel to the restrained roll

axis of the platform. Alternatively, the yaw torque could be measured during the measurement of I_{xx} (roll) to determine the same roll-yaw POI.

3. TEST CONDITIONS.

3.1 Test Item.

a. Vehicle Configuration.

(1) Establish and record the payload configuration for the vehicle, including any crew weight (and their positions), basic issue items (BII), and armor packages. Make sure all on-board items are secured from movement.

(2) All vehicle fluids should be checked and filled if necessary to minimize effects of weight shift. This is especially important for fuel tanks and any ballast tanks. If the tank cannot be completely filled for any reason, it should be emptied to avoid fluid slosh. Tank conditions (empty or full) and locations shall be reported.

(3) Set the tire pressures to their desired operating condition and record the values.

(4) Ensure that the desired suspension ride height conditions are set. When possible, lock out the suspension. Carefully measure the vehicle's ride height at selected reference points before and after restraining to the MOI platform.

(5) Clean the vehicle of any accumulated dirt/mud.

(6) Ensure that all the necessary vehicle characteristic data are recorded.

b. Acclimate the test vehicle to the facility's ambient temperature.

c. Minimum Weight. Ensure the test vehicle weighs at least 2 percent of the facility's maximum capacity[7]. In practice, it is recommended the test vehicle weighs at least approximately 75 percent of the carrier platform's weight. This limitation is due to the influence of the platform's weight on overall measurement error.

d. MOI Limits. Make sure that the vehicle's anticipated moments of inertia values are within the capabilities of the facility. MOI accuracy is reduced when the vehicle's MOI is less than that of the platform (tare) MOI about the axis of interest[10]. For example, the original IPMD facility had a reported platform tare roll inertia three times that of the typical small vehicle they were testing, which contributed to over half of the measurement uncertainty for that vehicle class[11].

e. Vehicle Location on the Platform.

(1) The vehicle is initially positioned on the platform with the vehicle CG (longitudinal and lateral) aligned with the platform centerlines. The vehicle is considered

aligned with the platform when the vehicle's CG offset relative to the pivot axis vertical projection on the platform is within 1% of the expected radius of gyration of the vehicle about the axis of interest, as shown in Equation Set 1:

$$CG\ Offset_{long} = H_v \cdot \tan(\theta_{p,long}) \leq 0.01K_y$$
$$CG\ Offset_{lat} = H_v \cdot \tan(\theta_{p,lat}) \leq 0.01K_x \qquad \textit{(Equation Set 1)}$$

where:

$CG\ Offset_{long}$ = Vehicle CG offset in the longitudinal direction (in.)
$CG\ Offset_{lat}$ = Vehicle CG offset in the lateral direction (in.)
H_V = Vehicle vertical CG component from pivot axis (in.)
$\theta_{p,long}$ = Platform static equilibrium angle in the longitudinal direction (deg, rad)
$\theta_{p,lat}$ = Platform static equilibrium angle in the lateral direction (deg, rad)
K_Y = Vehicle radius of gyration about the vehicle's centroidal pitch axis (in.)
K_X = Vehicle radius of gyration about the vehicle's centroidal roll axis (in.)

(2) Record the vehicle's position on the platform immediately prior to testing.

Note: CG offsets meeting this criterion would contribute only 0.01% error in the final MOI, which follows from the Parallel Axis Theorem's squared-distance shift[12]. Larger positional offsets may be acceptable if the vehicle is especially hard to maneuver. The radius of gyration can be initially estimated using techniques shown in Paragraph 4.1.b, but for typical large vehicle CG heights and platform pivot locations, if the platform is within 0.2 degrees of level with the vehicle in place, the criterion should be satisfied[3]. Vehicle position on the platform becomes especially critical if measuring the roll-yaw POI, which is very sensitive to CG offsets[13].

f. Vehicle Restraints. Restrain the test vehicle's sprung mass to the test platform to minimize vehicle dynamic movement during testing. Typically, some form of adjustable-height blocks should be used to restrain the vehicle's chassis from pitching and heaving motion, with straps constraining the vehicle against the blocks. Straps may also be used to restrain longitudinal, lateral, and yaw motion. Consideration of restraint selection should be given to make it easier to calculate the tare MOI with the restraints in their proper positions. Any restraint equipment utilized should be weighed and the position on the platform recorded.

Note: It was shown in reference number 5 that the error contributions to the MOI about the pitch and roll axes for unrestrained heavy-class vehicles were dominated by the motion of the vehicle in relation to the moving platform.

3.2 Test Fixture.

a. The vehicle-supporting platform should be symmetrical in design about the test axis of interest to eliminate the products of inertia in the tare MOI.

b. Platform Levelness. The platform's empty, static equilibrium position should be checked via inclinometers to verify its levelness. Check for suspension link binding before loading the test vehicle.

c. Platform Stiffness. When loaded with the test vehicle, it is recommended that the platform deflection is less than 0.25 inches at the vehicle's CG (as measured from the pivot when compared to the unloaded platform). Significant measurement errors are introduced when the deflection is greater than this, although corrections for deflection can be made.

d. Platform Weight. The lighter the platform, the better the overall measurement accuracy (i.e., use as light a platform as possible for the vehicle under test while maintaining the necessary platform stiffness).

e. Pivot Location. The MOI platform's pivot height, whether for a stable (hanging) or unstable (inverted) compound pendulum is very important for measurement accuracy. For any given vehicle, there is an optimal pivot height that minimizes the MOI measurement error. For heavy vehicles tested with stable pendulums, experience has shown that a pivot height approximately one meter above the vehicle CG works well[6]. Another rule of thumb is to place the pivot height at the vehicle roofline. In general (except for yaw pivots), the pivot height should be adjusted so that it is located at a height between the test vehicle's vertical CG and the vehicle's radius of gyration about the axis in question[2,14]. Practical consideration should also be given to the fact that having a faster swing rate in the case of a hanging pendulum with shorter suspension links increases the relative error in the period measurement and increases vehicle motion with respect to the platform[6].

f. Pivot Bearing Damping.

(1) The use of low friction pivot bearings, whether heavy-duty roller bearings for hanging pendulums or high capacity hydrostatic or air bearings for inverted or torsional pendulums, is critical since all current pendulum models assume negligible friction so that linearization techniques can be utilized. Acceptable levels of damping may vary depending on the facility's desired measurement uncertainty, but as a general rule, the damping error is insignificant if it takes more than 50 oscillations for the platform's swing amplitude to decay by a factor of ten[12]. This implies a damping ratio, ζ
(Even if it took only 10 swings for the platform's amplitude to decline by a factor of ten, the damping-related error would be 0.134%, which may be acceptable in some instances.)

(2) The log decrement method can be used to determine the damping ratio, as shown in Equation Set 2:

$$d = \frac{1}{n} ln\left(\frac{a_0}{a_n}\right)$$
$$\zeta = \left(1 + \left(\frac{2\pi}{d}\right)^2\right)^{-0.5} \qquad \textit{(Equation Set 2)}$$

where:

d = Log decrement (unit-less)
n = Number of oscillations
a_0 = Initial platform swing amplitude (deg, rad)
a_n = Swing amplitude after n periods (deg, rad)
ζ = Damping ratio (unit-less)

(3) The damping ratio can also be determined using a damped-sine curve fit method (see Paragraph 5.2.a(2)(a)) if the oscillation data are digitally recorded.

(4) The damping-related error is then ζ^2 and the corrected moment of inertia is the measured MOI multiplied by $(1-\zeta^2)$. (Note: This correction only needs to be applied when the oscillation period is measured directly via a timing device or not already corrected to the natural frequency from the damped frequency.)

g. Ambient Conditions. The ambient conditions should be controlled during testing, including temperature, wind speed, and ground vibration. It is recommended that the test fixture is in an enclosed structure with regulated temperature.

4. TEST PROCEDURES.

For the following procedures, it is assumed that the vehicle and platform CG locations are known. TOP 02-2-800[15] and International Organization for Standardization (ISO) / Committee Draft (CD) 19380 Heavy commercial vehicles and buses- Centre of gravity measurements- Tilt-table, Axle lift, and Stable pendulum test methods[16] provide guidance on CG measurement.

4.1 Stable, Hanging Pendulum I_{xx} and I_{yy} Moments of Inertia Measurements.

a. Record all pertinent physical dimensions and weights of the test vehicle and platform, including any restraints used.

b. Set the proper pivot height for the platform/vehicle combination to be tested according to the guidance described in Paragraph 3.2.e. If needed, the vehicle’s radius of gyration about the axis under test can be estimated using the correlations described in the SAE Technical Paper 960896[16]. The technique is based on a modified version of the textbook definition of MOI for a solid hexahedral shape and the overall vehicle dimensions, as shown in Equation 3.

$$K \cong \sqrt{\frac{1}{K_G}(a^2+b^2)} \qquad (Equation\ 3)$$

where:

K = Radius of gyration (in.)

K_G = Geometric constant - for pitch=13.6, for roll=13.7, for yaw=13.8 (unit-less)
a = Characteristic vehicle dimension - for pitch & yaw=length, for roll=width (in.)
b = Characteristic vehicle dimension - for pitch & roll=height, for yaw=width (in.)

c. Oscillation Procedure. The maximum platform oscillation amplitude should be in the range of 2.5 to 3.5 degrees for large vehicles with high CGs[6]. (See justification in Appendix B.) The use of a stationary indicator of the platform's initial amplitude (or a physical stop which can swing out of the way once platform motion has started) is recommended to achieve repeatable oscillation amplitudes. Manually apply an even force to the platform to start the excitation motion about the desired pivot axis without causing platform motion about the orthogonal axes. Allow a few oscillations to occur before recording data to allow any platform transient motion to steady out. Perform 10 test runs with a minimum of 10 periods per run for each test configuration, settling the platform and zeroing electronic instrumentation between runs. This method allows observation of oscillation period variability. Also, averaging the period measurements improves the mean period's effective resolution by a combined factor of the square root of the number of periods per run multiplied by the number of runs (in this case, the combined factor is ten)[18].

d. Record the oscillation period of the empty platform (tare), with any necessary restraints in position and the instrumentation properly located, using the oscillation procedure described previously. Always perform a platform MOI tare. Do not assume past tares are applicable, as test condition variances occur, such as changes in fixturing/restraining devices, pivot height, and environmental conditions (temperature, air pressure).

e. Position the test vehicle on the platform as described in Paragraph 3.1.e. Restrain the vehicle as described in Paragraph 3.1.f. Position the instrumentation as required.

f. Measure all necessary dimensions such as vehicle location, pivot height, restraint locations, etc.

g. Record the oscillation period for the platform/vehicle system with necessary restraints, using the oscillation procedure described in Paragraph 4.1.c.

h. Reposition the platform suspension links and reconfigure the instrumentation to test the next orthogonal axis as needed, repeating the steps in Paragraphs 4.1.b through 4.1.g.

i. Calculate the platform and vehicle MOI's utilizing the procedures described in Paragraph 5.2.

4.2 Multi-Filar Torsional Pendulum I_{zz} Moments of Inertia Measurements.

This type of hanging pendulum uses gravity and the intrinsic torsional spring force in twisting cables as the restoring forces for measurement of the MOI about a vehicle's yaw axis.

a. Record all pertinent physical dimensions and weights of the test vehicle and platform, including any restraints used.

b. Replace the platform suspension links used for I_{xx} and I_{yy} measurements with four axially stiff, uniform, and torsionally flexible vertical links (e.g., solid wire rope is best to minimize damping) of equal length. Position the links equidistant from the platform's centroid. It is recommended that the vertical link length be chosen so that the system's oscillation frequency is approximately 0.5 Hz. (Practically, this works out to a length of approximately 2-3 meters for large vehicles.) The vertical links should be attached to the platform and supporting structure in such a manner as to minimize backlash in the connections as the platform reaches peak amplitude and reverses course.

c. Record the oscillation period of the empty platform (tare), with any necessary restraints in position and instrumentation properly located, using the oscillation procedure described in Paragraph 4.1.c. It is recommended that platform yaw motion is excited and controlled in some manner to achieve pure rotational oscillations. Some oscillation aberrations, such as platform tilt angle error, can be averaged out utilizing multiple (three are recommended) and opposing timing locations. Platform precessional movement errors can be averaged out by timing the oscillations over an integer number of pendulum precessions[9,19]. Always perform a platform MOI tare. Do not assume past tares are applicable, as test condition variances occur, such as changes in fixturing/restraining devices, link length, and environmental conditions (temperature, air pressure).

d. Position the test vehicle on the platform as described in Paragraph 3.1.e. Restrain the vehicle as described in Paragraph 3.1.f. Position the instrumentation as required.

e. Measure all the necessary dimensions such as vehicle location, restraint locations, etc.

f. Record the oscillation period for the platform/vehicle system with necessary restraints, using the oscillation procedure described in Paragraph 4.1.c.

g. Calculate the platform and vehicle MOI's utilizing the procedures described in Paragraph 5.2.

4.3 Unstable, Inverted Pendulum I_{xx} and I_{yy} Moments of Inertia Measurements.

This type of pendulum consists of a platform with the pivot axis below the platform and with linear springs providing the restoring (stabilizing) force.

a. Record all pertinent physical dimensions and weights of the test vehicle and platform, including any restraints used.

b. Reconfigure the platform connection links, bearings, reaction springs, and instrumentation for the axis under test. It is recommended that the reaction springs' rate is selected to achieve a platform/vehicle system oscillation frequency of approximately 0.5 Hz[5].

c. Set the proper pivot height (if adjustable) for the platform/vehicle combination to be tested following the recommendations described in Paragraph 4.1.b.

d. Record the oscillation period of the empty platform (tare), with any necessary restraints in position and instrumentation properly located, using the oscillation procedure described in Paragraph 4.1.c. Always perform a platform MOI tare. Do not assume past tares are applicable, as test condition variances occur, such as changes in fixturing/restraining devices, pivot height, spring rates, and environmental conditions (temperature, air pressure).

e. Position the test vehicle on the platform as described in Paragraph 3.1.e. Restrain the vehicle as described in Paragraph 3.1.f. Position the instrumentation as required.

f. Measure all the necessary dimensions such as vehicle location, restraint locations, etc.

g. Record the oscillation period for the platform/vehicle system with necessary restraints, using the oscillation procedure described in Paragraph 4.1.c.

h. Calculate the platform and vehicle MOI's utilizing the procedures described in Paragraph 5.2.

4.4 Torsional Pendulum I_{zz} Moments of Inertia Measurements.

This type of pendulum consists of a platform with the pivot axis oriented vertically and centered with the platform dimensions. Linear springs provide the restoring (stabilizing) force.

a. Record all pertinent physical dimensions and weights of the test vehicle and platform, including any restraints used.

b. Reconfigure the platform connection links, bearings, reaction springs, and instrumentation for the axis under test. It is recommended that the reaction springs' rate is selected to achieve a platform/vehicle system oscillation frequency of approximately 0.5 Hz[5].

c. Record the oscillation period of the empty platform (tare), with any necessary restraints in position and instrumentation properly located, using the oscillation procedure described in Paragraph 4.1.c. Always perform a platform MOI tare. Do not assume past tares are applicable, as test condition variances occur, such as changes in fixturing/restraining devices, spring rates, and environmental conditions (temperature, air pressure).

d. Position the test vehicle on the platform as described in Paragraph 3.1.e. Restrain the vehicle as described in Paragraph 3.1.f. Position the instrumentation as required.

e. Measure all the necessary dimensions such as vehicle location, restraint locations, etc.

f. Record the oscillation period of the platform/vehicle system with necessary restraints, using the oscillation procedure described in Paragraph 4.1.c.

g. Calculate the platform and vehicle MOI's utilizing procedures in Paragraph 5.2.

4.5 I_{xz} Product of Inertia Measurements.

Measure the product of inertia I_{xz} using a torsional pendulum fixture (described in Paragraph 4.4) with an integrated, constrained roll axis (referring to the fixture). The roll constraint shall be stiff and includes a load cell(s) to measure the roll couple induced during yaw motion. This test is generally performed simultaneously with the yaw MOI measurement (Paragraph 4.4). The same procedures are followed, except that the roll couple is also measured during the test.

4.6 Typical Full Test Sequence.

The sequence listed in Table 4 minimizes placement and removal of the vehicle on the platform, thus reducing test time and repeatability errors.

TABLE 4. TYPICAL MOI TEST SEQUENCE

SEQUENCE OF TESTS
1. Platform tare MOI about pitch axis (include necessary restraints)
2. Vehicle MOI about pitch axis
3. Vehicle MOI about roll axis
4. Vehicle MOI about yaw axis
5. Platform tare MOI about yaw axis (include necessary restraints)
6. Platform tare MOI about roll axis (include necessary restraints)
7. Products of Inertia vehicle setups as required (if measured separately)

5. DATA REQUIRED.

5.1 Data and Measurements Required.

a. Weight and dimensions of the vehicle, N (lbf).

b. Weights and dimension of the platform, N (lbf).

c. Weights and positions of restraint hardware, N (lbf) and mm (in.).

d. Vehicle test configuration:

(1) Vehicle suspension setting.

(2) Tire pressure.

(3) Payload and ballast weights and locations.

(4) Fuel tank configuration, capacity, and location.

e. Position of vehicle on platform relative to centerline datum, mm (in.).

f. Pivot axis location relative to platform reference plane, mm (in.).

g. Oscillation period, millisecond (msec).

h. Reaction spring positions and rates, mm (in.) and N/mm (lb/in.).

i. Multi-filar pendulum link (or cable) lengths and positions relative to platform datum, mm (in.).

j. Roll axis load cell force, N (lbf).

k. Seat positions (in.).

5.2 Data Processing.

a. Methods for Finding the Oscillation Period.

(1) Discrete Timing. Discrete oscillation period measurements are averaged for the test trials performed. Ensure that full periods are measured.

(2) Continuous Timing. When continuous time measurements of platform motion are recorded, the oscillation period can be determined analytically. Some common methods are described below.

(a) Damped-Sine Curve Fit. Of the methods described, this approach is perhaps the most accurate. The platform oscillation angle or rate data are fit to a curve given by Equation 4 using a non-linear sum-squared error minimization optimization to find the coefficients. This method has excellent inter-test run repeatability (even with noisy data), makes full use of the data collected, and increases the effective resolution of the period compared to discretely timing the same number of periods[20].

$$\theta(t) = Offset + \theta_i \cdot e^{-\zeta(2\pi f\sqrt{1-\zeta^2})t} \cdot \sin\left(\left(2\pi f\sqrt{1-\zeta^2}\right)t - \phi\right) \quad \textit{(Equation 4)}$$

where:

$\theta(t)$ = Time-varying platform swing amplitude (deg)
$Offset$ = Platform equilibrium (mean) position (deg)
θ_i = Initial platform amplitude (deg)
ζ = Damping ratio (unit-less)
f = System natural frequency (Hz)
t = Time (sec)
ϕ = Phase angle (rad)

Note: See Appendix C for Matlab*** scripts implementing this technique.

(b) Fourier Analysis. A Fast Fourier Transform (FFT) analysis of the results may be used to estimate the oscillation period. However, this approach may not provide enough frequency resolution to produce an accurate result due to limitations on sampling rates and FFT block size. There is a technique[21] that increases the resolution of the FFT around the frequency range of interest via filtering and decimating the data. This method also produces good repeatability and is good for processing noisy data.

(c) Zero-Crossings. The period of oscillation can also be determined by observing zero-crossings of the angle or rate data. Data interpolation is used to establish the crossing times. This procedure may be problematic with noisy data signals or improperly filtered data.

b. Method to Calculate the Vehicle Motion Ratio. The MOI calculations that follow correct for vehicle displacement relative to the platform using the sensors described in paragraph 2.2.c(2). Vehicle displacement and platform angle measurements (versus time) may be fitted to damped sine curves (Equation 4) to smooth the data. Also, platform rate results may need to be integrated to yield angle results. Plot vehicle displacement results as the ordinate and platform angle results as the abscissa. Perform a linear least squares fit of the plot and compute the slope (units of ft/rad). The slope is then used as the motion ratio parameter R_p (units of ft/rad) in Equation 5 for the pitch and roll directions and R_z (units of rad/rad) in Equation 7 for the yaw direction. This step eliminates any time lag between the vehicle and platform motion if needed.

c. Methods to Calculate MOI and POI. A 2-DOF vehicle model is used to compute the MOI's, largely due to tire and suspension compliance between the vehicle's sprung mass and the platform. Use of the traditional 1-DOF model has been shown to produce calculation errors of 2-10%[3]. Simplifications made with the 2-DOF model include: assuming minimal system damping, using small angle approximations for linearization, and ignoring the second and higher vibrational modes (5 Hz and higher)[3].

Note: Application of the local gravity constant (rather than the universal constant) in the following calculations is important for accuracy. Local gravity constant data can be found via regional United States Geological Survey (USGS) reports[22] or estimation based on the International Gravity Formula and the Free Air Correction, which account for local latitude and altitude[23]. For ATC's facility, local g=9.80119 m/s^2, corrected to local altitude from a nearby gravity station and reconciled with a geodetic station located at the U.S Army Aberdeen Proving Ground[22,24].

*** The use of brand names does not constitute endorsement by the Army or any other agency of the Federal Government, nor does it imply that it is best suited for its intended application.

(1) Stable, Hanging Pendulum I_{xx} and I_{yy} MOI Calculations. Equation 5 is used to compute the vehicle's roll (I_{xx}) and pitch (I_{yy}) inertias. This equation includes corrections for the vehicle's relative displacement motion[3,5,and 6]:

$$I_v = \frac{T_t^2}{4\pi^2}\left[\left(W_p + W_v\right)H_T - R_p W_v\right] + R_p \frac{W_v}{g} H_v - I_p - \frac{W_v}{g} H_v^2 \qquad \textit{(Equation 5)}$$

where:

I_v = Moment of inertia about the vehicle's centroidal axis of interest (lbf-ft-s^2)
T_t = Total system period of oscillation (sec)
W_p = Weight of the platform (plus restraints) (lbf)
W_v = Weight of the vehicle (lbf)
H_T = System (platform and vehicle) CG height below pivot axis (ft)
R_p = Magnitude of ratio of vehicle longitudinal motion (for pitch inertia) or lateral motion (for roll inertia) to platform rotational motion (ft/rad)
H_v = Vehicle CG height below pivot axis (ft)
I_p = Moment of inertia of the platform (plus restraints) about the pivot axis, as defined below (lbf-ft-s^2)
g = Local gravity constant (ft/s^2)

$$I_p = \frac{T_p^2}{4\pi^2} W_p H_p \qquad \textit{(Equation 6)}$$

where:

T_p = Platform (plus restraints) period of oscillation (sec)
H_p = Platform (plus restraints) CG height below pivot axis (ft)

(2) Multi-Filar Torsional Pendulum I_{zz} MOI Calculations. Equation 7 is used to compute the vehicle's yaw (I_{zz}) inertia and contains a correction for the vehicle's relative motion[5,6, and 14]:

$$I_v = \frac{R_z\left[\frac{(W_p+W_v)R^2 T_t^2}{4\pi^2 L} - I_p\right]}{R_z + 1} \qquad \textit{(Equation 7)}$$

where:

I_v = Moment of inertia about the vehicle's centroidal yaw axis (lbf-ft-s^2)
W_p = Weight of the platform (plus restraints) (lbf)
W_v = Weight of the vehicle (lbf)
R = Radial (perpendicular) distance from the platform yaw axis to any of the vertical support wire centerlines (ft)
T_t = Total system period of oscillation (sec)

L = Length of the vertical support wires (from the platform attachment point to the upper support structure attachment point) (ft)

I_p = Moment of inertia of the platform (plus restraints) about the yaw axis, as defined below (lbf-ft-s^2)

R_z = Ratio of platform yaw motion to relative yaw motion between the vehicle and platform, as defined below (rad/rad)

$$I_p = \frac{W_p R^2 T_p^2}{4\pi^2 L} \qquad \textit{(Equation 8)}$$

where:

T_p = Platform (plus restraints) period of oscillation (sec)

$$R_z = \left|\frac{\psi_p}{\psi_p - \psi_v}\right| \qquad \textit{(Equation 9)}$$

where:

ψ_p = Platform yaw amplitude (rad)
ψ_v = Vehicle yaw amplitude (rad)
(Note that the quantity (ψ_p - ψ_v) is measured directly)

Figure 3 shows the pertinent dimensions needed for the yaw calculations on the platform.

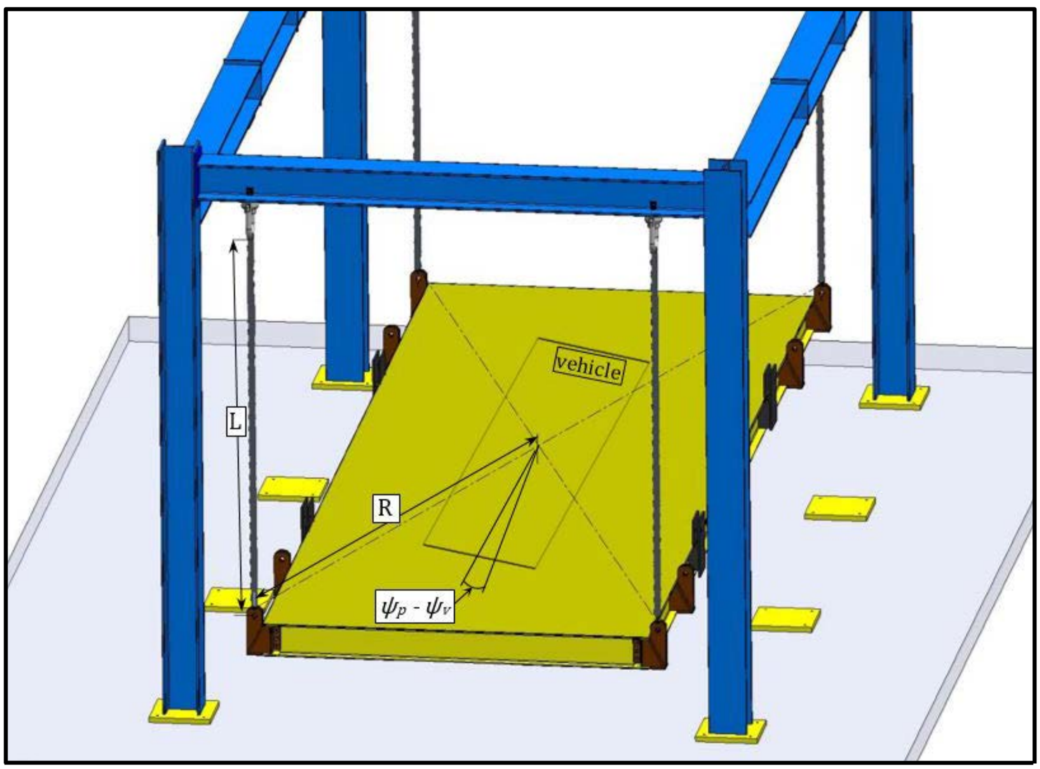

Figure 3. Multi-filar torsional pendulum setup showing dynamically-displaced vehicle outline and pertinent dimensions.

(3) Unstable, Inverted Pendulum I_{xx} and I_{yy} MOI Calculations. Equation 10 is used to compute the vehicle's roll (I_{xx}) and pitch (I_{yy}) inertias and contains corrections for the vehicle's relative displacement motion[3, 5, 6, and 25].

$$I_v = \frac{T_t^2}{4\pi^2}\left[K_u L_u^2 - \left(W_p + W_v\right)H_T - R_p W_v\right] + R_p \frac{W_v}{g} H_v - I_p - \frac{W_v}{g} H_v^2 \qquad \textit{(Equation 10)}$$

where:

I_V = Moment of inertia about the vehicle's centroidal axis of interest (lbf-ft-s^2)
T_t = Total system period of oscillation (sec)
K_u = Reaction spring stiffness (lbf/ft)
L_u = Moment arm length from reaction spring to pivot (ft)
W_p = Weight of the platform (plus restraints) (lbf)
W_V = Weight of the vehicle (lbf)
H_T = System (platform and vehicle) CG height from pivot axis (ft)
R_p = Magnitude of ratio of vehicle longitudinal motion (for pitch inertia) or lateral motion (for roll inertia) to platform rotational motion (ft/rad)
H_V = Vehicle CG height from pivot axis (ft)
I_p = Moment of inertia of the platform (plus restraints) about the pivot axis, as defined below (lbf-ft-s^2)
g = Local gravity constant (ft/s^2)

$$I_p = \frac{T_p^2}{4\pi^2}\left[K_u L_u^2 - W_p H_p\right] \qquad \textit{(Equation 11)}$$

where:

T_p = Platform (plus restraints) period of oscillation (sec)
H_p = Platform (plus restraints) CG height from pivot axis (ft)

(4) Torsional Pendulum I_{zz} MOI Calculations. Equation 12 is used to compute the vehicle's yaw (I_{zz}) inertia and contains a correction for the vehicle's relative motion[5,6].

$$I_v = \frac{R_z\left[\frac{T_t^2 K_p}{4\pi^2} - I_p\right]}{R_z + 1} \qquad \textit{(Equation 12)}$$

where:

I_V = Moment of inertia about the vehicle's centroidal yaw axis (lbf-ft-s^2)
T_t = System period of oscillation (sec)
K_p = Stiffness of the spring between platform and ground (lbf-ft/rad)
I_p = Moment of inertia of the platform (plus restraints) about the yaw axis, as defined below (lbf-ft-s^2)

R_z = Ratio of platform yaw motion to relative yaw motion between the vehicle and platform, as defined in Equation 9 (rad/rad)

$$I_p = \frac{T_p^2 K_p}{4\pi^2} \qquad \textit{(Equation 13)}$$

where:

T_p = Platform (plus restraints) period of oscillation (sec)

(5) Roll-Yaw (I_{xz}) Product of Inertia Calculations. Equation 14 is used to compute the vehicle's roll-yaw (I_{xz}) product of inertia[6].

$$I_{xz} = -\frac{T_x}{\ddot{\psi}_p} \qquad \textit{(Equation 14)}$$

where:

I_{xz} = Roll-yaw product of inertia about the vehicle's centroid (lbf-ft-s^2)
T_x = Roll reaction moment (lbf-ft)
$\ddot{\psi}_p$ = Platform yaw angular acceleration at maximum displacement (rad/s^2)

The yaw acceleration amplitude is taken most conveniently from a damped-sine curve fit of the time-dependent platform yaw angle such that the acceleration amplitude becomes the curve-fitted initial platform amplitude θ_i (rad) multiplied by the square of the curve-fitted oscillation frequency (rad/s). The roll reaction moment needed for Equation 14 is calculated as follows in Equation 15, accounting for the vehicle's longitudinal offset on the platform[6]:

$$T_x = T_{meas} - \frac{(W_p + W_v)}{g} H_T O_x \ddot{\psi}_p \qquad \textit{(Equation 15)}$$

where:

T_{meas} = Roll torque amplitude measured during the yaw test (lbf-ft)
H_T = System (platform and vehicle) CG to roll pivot axis (ft)
O_x = Vehicle CG offset in the longitudinal direction, as defined in equation 1 (ft)
W_p, W_v, g, and $\ddot{\psi}_p$ are as defined previously

It should be noted that the calculation of I_{xz} above assumes that the platform (with restraints) roll-yaw POI is zero. If the platform is not symmetric in the x-z plane as specified, its POI must be determined from empty platform testing and subtracted from Equation 14.

(6) Parallel Axis Theorem Translations. To translate any MOI result from the vehicle's CG to an arbitrary, parallel axis, use Equation 16.

$$I_{arb} = I_{CG} + \frac{W_v}{g} r^2 \qquad \textit{(Equation 16)}$$

where:

I_{arb} = New MOI about the translated axis (lbf-ft-s^2)
I_{CG} = Vehicle MOI about the centroidal axis of interest (lbf-ft-s^2)
r = Perpendicular distance between the two axes (ft)
W_v, g are as defined previously

(7) Error/Uncertainty Estimation. The Law of Propagation of Uncertainty is the appropriate method to use for MOI error estimation because it applies to multi-parameter, single-sample experiments[4]. To compute the overall system error, one must describe the system (i.e., the vehicle's MOI or POI) as a function of the individual measured parameters (i.e., weight, dimensions, period, etc.). Then the system error can be determined using Equation 17.

$$E_{rss} = \sqrt{\left(\Delta u_1 \cdot \frac{\partial N}{\partial u_1}\right)^2 + \left(\Delta u_2 \cdot \frac{\partial N}{\partial u_2}\right)^2 + \cdots + \left(\Delta u_n \cdot \frac{\partial N}{\partial u_n}\right)^2} \qquad \textit{(Equation 17)}$$

where:

E_{rss} = Root-sum square system (MOI, POI) error
N is the system function of all the u_i variables (weight, dimensions, period, etc.)
Δu_i = Single standard deviation error (uncertainty) of the u_i^{th} variable

Once each error component is calculated, its relative contribution to the overall error can be determined and analyzed to discover where system improvements can be made. If confidence intervals need to be calculated, the 'k' factor from the Student's t distribution should be used in place of the normal distribution factor due to the use of a small population size based on the number of measured parameters and tests conducted[26]. In error analysis, keep in mind the Weak Link Principle, which states that the final measurement can't be any better than the most error-prone instrument used in the process[27].

(d) Radius of Gyration Calculation. The radius of gyration of a vehicle, about the centroidal axis of interest, is the distance from that axis at which all of the mass of the vehicle could be concentrated without changing its moment of inertia. The radius is calculated using Equation 18.

$$K = \sqrt{\frac{I_v}{W_v/g}} \qquad \textit{(Equation 18)}$$

where:

K = Radius of gyration (ft)
I_v = Moment of inertia about the vehicle's centroidal axis of interest (lbf-ft-s^2)
W_v = Weight of the vehicle (lbf)
g = Local gravity constant (ft/s^2)

Once the actual radii of gyration are found for the vehicle, they can be compared to the estimates calculated in Paragraph 4.1.b to ensure the platform pivot height criterion was met (see Paragraph 3.2.e) and the vehicle CG's static position on the platform criterion was met (see Paragraph 3.1.e) for error minimization purposes.

5.3 Calculation Checks/Rules of Thumb[10].

a. The MOI about one axis cannot be greater than the sum of the MOI's about the other two orthogonal axes.

b. The radius of gyration about one axis must be less than the longest dimension at right angles to the axis.

c. There are 4 inequality rules relating MOI's and POI's, but for a vehicle with basic lateral symmetry, only I_{xz} is relevant. Therefore, Equation 19 must hold:

$$\frac{4\, I_{xz}^2}{I_{yy}^2 - (I_{xx} - I_{zz})^2} \leq 1 \qquad \textit{(Equation 19)}$$

where:
I_{xz} = Roll-yaw product of inertia (lbf-ft-s^2)
I_{yy} = Moment of inertia about the vehicle's centroidal pitch axis (lbf-ft-s^2)
I_{xx} = Moment of inertia about the vehicle's centroidal roll axis (lbf-ft-s^2)
I_{zz} = Moment of inertia about the vehicle's centroidal yaw axis (lbf-ft-s^2)

6. PRESENTATION OF DATA.

a. The required data should be presented in a narrative or tabular format, as appropriate. The data should include, as needed:

(1) I_{xx} - Mass moment of inertia about the vehicle's centroidal longitudinal (roll) axis.

(2) I_{yy} - Mass moment of inertia about the vehicle's centroidal lateral (pitch) axis.

(3) I_{zz} - Mass moment of inertia about the vehicle's centroidal vertical (yaw) axis.

(4) K_x - Radius of gyration about the vehicle's centroidal longitudinal (roll) axis.

(5) K_y - Radius of gyration about the vehicle's centroidal lateral (pitch) axis.

(6) K_z - Radius of gyration about the vehicle's centroidal vertical (yaw) axis.

(7) I_{xz} - Roll-yaw product of inertia about the vehicle's centroid.

b. Particular attention should be paid to keeping the calculated values' units of measure consistent. An estimate of each of the above value's measurement uncertainties and corresponding confidence level should be included.

c. Figure 4 shows a sample data form used for MOI testing.

WHEELED TEST VEHICLE DESCRIPTION

TEST PROJECT NO. **TEST COMPLETION DATE:**

VEHICLE			
Year:	Make:	Model:	
Type:	Usage:	Status:	
Serial Number:		Body Style:	
Registration No:		Odometer (mi):	Engine Hours:
WHEELBASE (in)			
OA:	Axle 1-2:	Axle 2-3:	Axle 3-4:
TRACK (in)			
Axle 1:	Axle 2:	Axle 3:	Axle 4:
GVWR (lb):		**GCWR (lb):**	
GAWR (lb)			
Axle 1:	Axle 2:	Axle 3:	Axle 4:
CURB WEIGHT (lb)	Total: 50,062		
L1: |R1:	L2: |R2:	L3: |R3:	L4: |R4:
POWERTRAIN			
Engine Type:	Orientation:	Configuration:	Engine Location:
Transmission Type:	Transmission Gears:	Options:	
Comments:			
SUSPENSION - AXLE 1			
Type:		Stabilizer Bar:	
Springs:		Shocks:	
Driven:		Differential:	
Features:		Shock Control:	
Comments:			
SUSPENSION - AXLE 2			
Type:		Stabilizer Bar:	
Springs:		Shocks:	
Driven:		Differential:	
Features:		Shock Control:	
Comments:			

Figure 4. Sample MOI data collection form.

APPENDIX A. ABBREVIATIONS.

ASME	American Society of Mechanical Engineers
ATC	U.S. Army Aberdeen Test Center
AVTP	Allied Vehicle Test Publication
BII	basic issue items
CD	Committee Draft
CG	center of gravity
cm	centimeter
DOF	degree of freedom
DSC	Dynamic Systems and Control
FFT	Fast Fourier Transform
ft	feet
ft/s^2	feet per second squared
HSRI	Highway Safety Research Institute
HVE	Human Vehicle Environment
Hz	Hertz
IEEE	Institute of Electrical and Electronics Engineers
IMU	Inertial Motion Unit
in.	inch
in/s^2	inches per second squared
IPMD	Inertial Parameter Measurement Device
ISO	International Organization for Standardization
kg	kilogram
lb	pound
lbf	pound force
lbm	pound mass
Matlab	matrix laboratory
m/s^2	meters per second squared
mm	millimeter
MOI	moments of inertia
msec	millisecond

APPENDIX A. ABBREVIATIONS.

N	Newton
NATO	North Atlantic Treaty Organization
NHTSA	National Highway Traffic Safety Administration
NRMM	NATO Reference Mobility Model
POI	product of inertia
SAE	Society of Automotive Engineers
SAWE	Society of Allied Weight Engineers
sec	second
TARDEC	U.S. Army Tank Automotive Research, Development and Engineering Center
TOP	Test Operations Procedure
USGS	United States Geological Survey
VIMF	Vehicle Inertia Measurement Facility
VIPER	Vehicle Inertial Properties Evaluation Rig
VIPER II	Vehicle Inertia Parameter Evaluation Rig

APPENDIX B. SMALL ANGLE REQUIREMENT.

a. While it is generally true that using an angle less than 5 degrees produces only a fractional error (0.127 %) when linearizing the pendulum equation of motion with the sine function approximation, $\sin\theta \cong \theta$, the error incurred produces oscillation period errors greater than the permissible measurement error of 0.001 sec when used on compound pendulum systems for large vehicles (where the pivot-to-CG length is about 1.2 m (4.0 ft) or longer).

b. The idealized pendulum period is defined as Equation B-1:

$$T_0 = 2\pi\sqrt{L/g} \qquad \textit{(Equation B-1)}$$

where:

T_0 = Ideal period
L = Pendulum length (pivot-to-CG)
g = Local gravity constant

c. The pendulum equation of motion can be solved by a perturbation expansion where the correction to the ideal period is given by a Taylor series expansion[28] (Equation B-2).

$$\frac{\Delta T}{T_0} = \sum_{n=1}^{\infty}\left(\frac{(2n)!}{2^{2n}(n!)^2}\right)^2 \sin^{2n}\left(\frac{\theta_0}{2}\right) = \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \ldots \qquad \textit{(Equation B-2)}$$

where:

ΔT = Period correction
θ_0 = Maximum oscillation amplitude (rad)

and the true period, $T = T_0 + \Delta T$.

d. The sine small angle approximation error at 5 degrees exceeds the 0.001 sec period correction threshold when the pendulum length exceeds 1.1 m (3.6 ft). At a pendulum length of 1.2 m (4.0 ft), the period correction is half the permissible error at an amplitude of 3.5 degrees, providing a comfortable margin for those instances when the pendulum length is longer.

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APPENDIX C. MATRIX LABORATORY (MATLAB) CODE FOR DATA PROCESSING.

```
% Maximizes R^2 value of damped sine fit to periodic (noisy) data &
% returns optimized coefficients & their standard errors.
% DPK Nov 2016

%% Change to data channels of interest or setup data import here-USER INPUT
timeI = Time;
%valuesI = TablePitchRate;
%valuesI = TableAccelX;
valuesI = TablePitchAngle;
%valuesI = TablePitchanglesensor;

% Set coefficient initial value guesses - USER INPUT
length_p = 170/12;                          % pendulum length (ft)
Fs = 1/(timeI(2)-timeI(1));                 % sample rate (Hz)
offset = nanmean(valuesI);                  % "zero" equals average data value
amp = abs(nanmax(valuesI))-offset;          % make sure amplitude is positive
freq = 1/(2*pi*sqrt(length_p/32.174));      % ideal pendulum frequency
% Calc approx phase shift based on location of first positive peak compared
% to simple sine wave.
% findpeaks arguments used to deal with noisy signals
[pks,locs] = findpeaks(valuesI,Fs,'MinPeakDistance',1/...
    (freq*4),'MinPeakHeight',0.5*amp,'NPeaks',3);
phase = 2*pi*freq*locs(find(pks==max(pks),1))-pi/2;
% try to keep phase +/-2Pi
if phase >= 2*pi || phase <= -2*pi
    phase = phase-fix(phase/pi)*pi;
end
damping = 0;                                % reset damping to zero
fprintf('\n Initial guess for Amplitude = %.15f',amp);
fprintf('\n Initial guess for Offset = %.15f',offset);
fprintf('\n Initial guess for Phase Shift = %.15f',phase);
fprintf('\n Initial guess for Frequency = %.15f',freq);
fprintf('\n Initial guess for Damping = %.15f',damping);
fprintf('\n');

AguessI = 0;
OguessI = 0;
PguessI = 0;
FguessI = 0;
DguessI = 0;

%% Loop to run new fit routine replacing initial coefficient guesses with
% the values from 'fminsearch' until the tolerance is reached.
% Assumes coefficients are of approx same order for tolerance, if not,
% base tol on smallest coefficient.
tol = 1e-9;                                 % coefficient change tolerance
n = 0;          % # of coefficient replacement loops needed for convergence
while abs(AguessI-amp)>tol || abs(OguessI-offset)>tol || abs(PguessI-...
        phase)>tol || abs(FguessI-freq)>tol || abs(DguessI-damping)>tol
% trying to keep amplitude non-negative while converging
AguessI = abs(amp);
OguessI = offset;
PguessI = phase;
```

APPENDIX C. MATRIX LABORATORY (MATLAB) CODE FOR DATA PROCESSING.

```
FguessI = freq;
DguessI = damping;

% fminsearch options - modify to your case as needed
options = optimset('MaxFunEvals', 1e10, 'MaxIter', 1e4, 'TolFun', 1e-9,...
    'TolX', 1e-9);

% ICoeffs=InputCoefficients=[Amplitude,Offset(vertical),Phase Shift,
% Frequency,Damping Ratio] of model that best fits.
% For function that returns R^2 value of the fit of the sine wave created
% from the input coefficients.
ICoeffs = fminsearch(...
    @(x)1-SinDampedOpt(x, timeI, valuesI),...
    [AguessI, OguessI, PguessI, FguessI, DguessI],...     % Starting point
    options);

% R^2 value of the final Input y model returned by fminsearch
IR2 = SinDampedOpt(ICoeffs, timeI, valuesI);

% set coefficients to new fit values
amp = ICoeffs(1);
offset = ICoeffs(2);
phase = ICoeffs(3);
freq = ICoeffs(4);
damping = ICoeffs(5);
f_damped = freq*sqrt(1-damping^2);

n = n+1;
% END coefficient LOOP
end
%% Outputs Section
% calculate fit function using new coefficients
y_fit = offset+amp*exp(-damping*timeI*freq*sqrt(1-damping^2)*2*pi).*...
    sin(timeI*freq*sqrt(1-damping^2)*2*pi-phase);

SSres = sum((valuesI-y_fit).^2);                % fit residuals
df = length(valuesI)-length(ICoeffs);           % degrees of freedom

% calculate analytical partial derivatives of damped sine WRT coefficients
% (b0=offset,b1=amp,b2=damping,b3=freq,b4=phase)
partial_b0 = ones(length(valuesI),1);
partial_b1 = exp(-damping*timeI*freq*sqrt(1-damping^2)*2*pi).*...
    sin(timeI*freq*sqrt(1-damping^2)*2*pi-phase);
partial_b2 = -(amp*2*pi*freq*timeI/sqrt(1-damping^2)).*...
    exp(-damping*timeI*freq*2*pi*sqrt(1-damping^2)).*...
    ((1-2*damping^2)*sin(timeI*freq*sqrt(1-damping^2)*2*pi-phase)+...
    damping*cos(timeI*freq*sqrt(1-damping^2)*2*pi));
partial_b3 = amp*2*pi*timeI.*exp(-damping*timeI*freq*sqrt(1-damping^2)*...
    2*pi).*(cos(timeI*freq*sqrt(1-damping^2)*2*pi-phase)*...
    sqrt(1-damping^2)-damping*sqrt(1-damping^2)*sin(timeI*freq*...
    sqrt(1-damping^2)*2*pi-phase));
partial_b4 = -amp*exp(-damping*timeI*freq*sqrt(1-damping^2)*2*pi).*...
    cos(timeI*freq*sqrt(1-damping^2)*2*pi-phase);
% consolidate partials into Jacobian matrix
J = [partial_b0,partial_b1,partial_b2,partial_b3,partial_b4];
```

APPENDIX C. MATRIX LABORATORY (MATLAB) CODE FOR DATA PROCESSING.

```
Vp = inv(J'*J);                          % coefficient covariance matrix
sigma_p = sqrt(SSres/df*diag(Vp));       % coefficient est. standard errors
% sqrt(RMS error) of fit:
sigma_y = sqrt(((valuesI-y_fit)'*(valuesI-y_fit))/(df+1));

fprintf('\n Final value for Amplitude = %.15f',amp);
fprintf('\n Standard Error of Amplitude = %.15f',sigma_p(2));
fprintf('\n Final value for Offset = %.15f',offset);
fprintf('\n Standard Error of Offset = %.15f',sigma_p(1));
fprintf('\n Final value for Phase Shift = %.15f',phase);
fprintf('\n Standard Error of Phase Shift = %.15f',sigma_p(5));
fprintf('\n Final value for Frequency = %.15f',freq);
fprintf('\n Standard Error of Frequency = %.15f',sigma_p(4));
fprintf('\n Final value for Damping = %.15f',damping);
fprintf('\n Standard Error of Damping = %.15f',sigma_p(3));
fprintf('\n Final value for Damped Freq = %.15f',f_damped);
fprintf('\n');
fprintf('\n Standard Error of Fit = %.15f',sigma_y);
fprintf('\n Final value for R^2 = %.15f',IR2);
fprintf('\n');
fprintf(...
'\n # of coefficient replacement loops needed for convergence = %d',n);
fprintf('\n');

% plot original & fit data to provide visual check of adequate fit
figure
plot(timeI,valuesI)
hold on
plot(timeI,y_fit)

% consolidate all coefficients, errors & R^2 to pipe to Excel as needed
params = [ICoeffs,f_damped;...
    sigma_p(2),sigma_p(1),sigma_p(5),sigma_p(4),sigma_p(3),sigma_p(4);...
    sigma_y,IR2,NaN,NaN,NaN,NaN]';
function R2 = SinDampedOpt(x, time, values)
% calculates traditional least-squares fit R^2 value for damped sine
% function and given input params.

a = x(1);
o = x(2);
p = x(3);
f = x(4);
d = x(5);

fi = o+a*exp(-d*time*f*sqrt(1-d^2)*2*pi).*sin(time*f*sqrt(1-d^2)*2*pi-p);
yi = values;


SStot = sum((yi-mean(yi)).^2);
SSres = sum((yi-fi).^2);
SSE = sum(abs((fi-yi).^2));

R2 = 1-(SSres/SStot);

end
```

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APPENDIX E. APPROVAL AUTHORITY.

CSTE-TM 3 August 2017

MEMORANDUM FOR

Commanders, All Test Centers
Technical Directors, All Test Centers
Directors, U.S. Army Evaluation Center
Commander, U.S. Army Operational Test Command

SUBJECT: Test Operations Procedure (TOP) 01-2-520 Moments of Inertia, Approved for Publication

1. TOP 01-2-520 Moments of Inertia, has been reviewed by the U.S. Army Test and Evaluation Command (ATEC) Test Centers, the U.S. Army Operational Test Command, and the U.S. Army Evaluation Center. All comments received during the formal coordination period have been adjudicated by the preparing agency. The scope of the document is as follows:

This document describes practical methods for the determination of the mass MOI of large, heavy military vehicles, typically about the centroidal axes. Moments of inertia are fundamental mass properties that provide information on a vehicle's mass distribution. The properties impact vehicle design and safety and are primary inputs to vehicle dynamics and mobility computer models, where MOI data for a vehicle's total, sprung, and unsprung masses are often required.

2. This document is approved for publication and will be posted to the Reference Library of the ATEC Vision Digital Library System (VDLS). The VDLS website can be accessed at https://vdls.atc.army.mil/.

3. Comments, suggestions, or questions on this document should be addressed to U.S. Army Test and Evaluation Command (CSTE-TM), 2202 Aberdeen Boulevard-Third Floor, Aberdeen Proving Ground, MD 21005-5001; or e-mailed to usarmy.apg.atec.mbx.atec-standards@mail.mil.

HUBNER.MICHAEL
.WINFRIED.104288
0147

MICHAEL W. HUBNER
Associate Director, Test Management Directorate (G9)

FOR

RAYMOND G. FONTAINE
Director, Test Management Directorate (G9)

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Forward comments, recommended changes, or any pertinent data which may be of use in improving this publication to the following address: Policy and Standardization Division (CSTE-TM), U.S. Army Test and Evaluation Command, 2202 Aberdeen Boulevard, Aberdeen Proving Ground, Maryland 21005-5001. Technical information may be obtained from the preparing activity: Automotive Directorate (TEDT-AT-AD), U.S. Army Aberdeen Test Center, 400 Colleran Road, Aberdeen Proving Ground, Maryland 21005. Additional copies can be requested through the following website: http://www.atec.army.mil/publications/topsindex.aspx, or through the Defense Technical Information Center, 8725 John J. Kingman Rd., STE 0944, Fort Belvoir, VA 22060-6218. This document is identified by the accession number (AD No.) printed on the first page.